MATEMATIKA II TKS 4003

TURUNAN PARSIAL
(Partial Derivative)

Turunan

Fungsi, $y(x)$	Turunan, y'	Fungsi, $y(x)$	Turunan, y'
Konstanta	0	$\sin^{-1}(ax+b)$	$\frac{a}{\sqrt{1-(ax+b)^2}}$
x^n	nx^{n-1}	$\cos^{-1}(ax+b)$	$\frac{-a}{\sqrt{1-(ax+b)^2}}$
e^x	e^x	$\tan^{-1}(ax+b)$	$\frac{a}{1+(ax+b)^2}$
e^{-x}	$-e^{-x}$	$\sinh(ax+b)$	$a\cosh(ax+b)$
e^{ax}	ae^{ax}	$\cosh(ax+b)$	$a\sinh(ax+b)$
$\ln x$	$\frac{1}{x}$	$\tanh(ax+b)$	$a\,\mathrm{sech}^2(ax+b)$
$\sin x$	$\cos x$	$\mathrm{cosech}(ax+b)$	$-a\,\mathrm{cosech}(ax+b)\coth(ax+b)$
$\cos x$	$-\sin x$	$\mathrm{sech}(ax+b)$	$-a\,\mathrm{sech}(ax+b)\tanh(ax+b)$
$\sin(ax+b)$	$a\cos(ax+b)$	$\coth(ax+b)$	$-a\,\mathrm{cosech}^2(ax+b)$
$\cos(ax+b)$	$-a\sin(ax+b)$	$\sinh^{-1}(ax+b)$	$\frac{a}{\sqrt{(ax+b)^2+1}}$
$\tan(ax+b)$	$a\sec^2(ax+b)$	$\cosh^{-1}(ax+b)$	$\frac{a}{\sqrt{(ax+b)^2-1}}$
$\mathrm{cosec}(ax+b)$	$-a\,\mathrm{cosec}(ax+b)\cot(ax+b)$	$\tanh^{-1}(ax+b)$	$\frac{a}{\sqrt{1-(ax+b)^2}}$
$\sec(ax+b)$	$a\sec(ax+b)\tan(ax+b)$		

SIFAT-SIFAT TURUNAN

Jika u dan v adalah fungsi dalam x, dan c adalah konstanta, maka berlaku

1. f(x) = u + v maka f '(x) = u' + v'
2. f(x) = u - v maka f '(x) = u'-v'
3. f(x) = c.u maka f '(x)=c.u'
4. f(x) = u.v maka f'(x) = u'v + uv'
5. $f(x) = \frac{u}{v}$ maka $f'(x) = \frac{u'v - uv'}{v^2}$

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Partial Derivative

- Jika ada ***f*** sbg fungsi dari satu variabel ***x*** →turunan pertama fungsi ***f*** hanya ***terhadap x*** dinotasikan sbg:

$$f' = f'(x) = \frac{\partial f}{\partial x}$$

- Jika ada fungsi ***f dari dua variabel***, maka ***turunan pertama fungsi f*** dapat dicari untuk kedua variabel tersebut. Masing-masing turunan disebut sebagai ***turunan parsial***.

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Jika ***f*** fungsi dua peubah, x dan y, maka:

- Turunan parsial ***f*** terhadap x, notasi: $\frac{\partial f(x,y)}{\partial x}$ atau $f_x(x,y)$

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$$

- Turunan parsial ***f*** terhadap ***y***, notasi: $\frac{\partial f(x,y)}{\partial y}$ atau $f_y(x,y)$

$$\frac{\partial f(x,y)}{\partial x} y = \lim_{\Delta x \to 0} \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y}$$

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CONTOH 1

Tentukan turunan parsial terhadap ***x*** dan turunan parsial terhadap ***y*** fungsi yang dirumuskan dengan ***f(x,y) = x²y + x + y + 1***. Selanjutnya tentukan turunan parsial ***f*** *terhadap* x *dan* y di titik (1,2)

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 y + (x+\Delta x) + y + 1 - \left(x^2 y + x + y + 1\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 y + 2x.\Delta x.y + (\Delta x)^2 y + x + \Delta x + y + 1 - \left(x^2 y + x + y + 1\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x.\Delta x.y + (\Delta x)^2 y + \Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} 2x.y + \Delta x.y + 1$$

$$= 2xy + 1$$

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$$\frac{\partial f(x,y)}{\partial y} = \lim_{\Delta x \to 0} \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y}$$

$$= \lim_{\Delta x \to 0} \frac{x^2(y+\Delta y) + x + (y+\Delta y) + 1 - (x^2 y + x + y + 1)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 y + x^2.\Delta y + x + y + \Delta y + 1 - (x^2 y + x + y + 1)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2.\Delta y + \Delta y}{\Delta y}$$

$$= x^2 + 1$$

- Turunan parsial ***f*** terhadap x di titik (1,2) adalah: $\frac{\partial f(1,2)}{\partial x} = 2(1)(2) + 1 = 5$
- Turunan parsial ***f*** terhadap y di titik (1,2) adalah: $\frac{\partial f(1,2)}{\partial y} = 1^2 + 1 = 2$

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Turunan parsial dari fungsi dua peubah ***f(x)***:

1. Jika ***f*** diturunkan terhadap peubah ***x*** maka ***y*** dianggap tetap/konstanta.
2. Jika ***f*** diturunkan terhadap peubah ***y*** maka ***x*** dianggap tetap/konstanta.

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CONTOH 2

Tentukan turunan parsial terhadap ***x*** dan turunan parsial terhadap ***y*** fungsi yang dirumuskan dengan ***f(x,y) = 3x⁴y² + xy² + 4y***

$$\frac{\partial f(x,y)}{\partial x} = 12x^3y^2 + y^2$$

$$\frac{\partial f(x,y)}{\partial y} = 6x^4y + 2xy + 4$$

LATIHAN SOAL 1

Tentukan turunan parsial terhadap **x, y dan z** dari fungsi berikut:

$$w = x^2 - xy + y^2 + 2yz + 2z^2 + z,$$

$$\frac{\partial w}{\partial x}, \quad \frac{\partial w}{\partial y}, \quad \text{and} \quad \frac{\partial w}{\partial z}.$$

....??

$$\frac{\partial w}{\partial x} = 2x - y$$

$$\frac{\partial w}{\partial y} = -x + 2y + 2z$$

$$\text{and} \quad \frac{\partial w}{\partial z} = 2y + 4z + 1$$

LATIHAN SOAL 2

Tentukan turunan parsial terhadap **x** dan **y** dari fungsi berikut:

$$f(x,y)=e^{xy}+y\,ln\,x$$

$$\frac{\partial f(x,y)}{\partial x}=f_x=y.e^{xy}+y.\frac{1}{x}$$
$$=ye^{xy}+\frac{y}{x}$$

$$\frac{\partial f(x,y)}{\partial y}=f_x=x.e^{xy}+1.\,ln\,x$$
$$=xe^{xy}+ln\,x$$

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LATIHAN SOAL 2

Tentukan turunan parsial terhadap **x** dan **y** dari fungsi berikut:

$$f(x,y)=\frac{2x+3y}{x-3y}$$

$$f(x,y)=\sin\left(\frac{x}{1+y}\right),\ \text{calculate}\ \frac{\partial f}{\partial x}\ \text{and}\ \frac{\partial f}{\partial y}.$$

$$f(x,y,z)=e^{xy}\,ln\,z$$

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Higher Derivative

Turunan Parsial Tingkat Tinggi - Higher Derivatives

- Turunan fungsi biasanya masih berupa fungsi yang dapat diturunkan lagi. Jadi dari suatu fungsi kita dapat mencari turunan tingkat satu, turunan tingkat dua dan seterusnya.

Jika z = f(x,y) maka:

$$(f_x)_x = f_{xx} = f_{11} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 z}{\partial x^2}$$

$$(f_x)_y = f_{xy} = f_{12} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y\, \partial x} = \frac{\partial^2 z}{\partial y\, \partial x}$$

$$(f_y)_x = f_{yx} = f_{21} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x\, \partial y} = \frac{\partial^2 z}{\partial x\, \partial y}$$

$$(f_y)_y = f_{yy} = f_{22} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 z}{\partial y^2}$$

Tentukan semua turunan parsial order 2 dari $w = x^3y^2 - xy^5$.

$$\frac{\partial w}{\partial x} = 3x^2y^2 - y^5$$

$$\frac{\partial w}{\partial y} = 2x^3y - 5xy^4$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x}\right) = 6xy^2$$

$$\frac{\partial^2 w}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial w}{\partial y}\right) = 2x^3 - 20xy^3$$

$$\frac{\partial^2 w}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial y}\right) = 6x^2y - 5y^4$$

$$\frac{\partial^2 w}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial w}{\partial y}\right) = 6x^2y - 5y^4$$

LATIHAN SOAL

Kerjakan semua contoh yang ada di soal bab pertama..!!

Mixed Partial Derivatives

- Note that $f_{xy} = f_{yx}$ in the preceding example, which is not just a coincidence.
- It turns out that $f_{xy} = f_{yx}$ for most functions that one meets in practice:

Clairaut's Theorem Suppose f is defined on a disk D that contains the point (a, b). If the functions f_{xy} and f_{yx} are both continuous on D, then

$$f_{xy}(a, b) = f_{yx}(a, b)$$

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Mixed Partial Derivatives

- Partial derivatives of order 3 or higher can also be defined. For instance,

$$f_{xyy} = (f_{xy})_y = \frac{\partial}{\partial y}\left(\frac{\partial^2 f}{\partial y\,\partial x}\right) = \frac{\partial^3 f}{\partial y^2\,\partial x}$$

and using Clairaut's Theorem we can show that $f_{xyy} = f_{yxy} = f_{yyx}$ if these functions are continuous.

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Contoh

- Calculate f_{xxyz} if $f(x, y, z) = \sin(3x + yz)$.
- Solution

$$f_x = 3\cos(3x + yz)$$

$$f_{xx} = -9\sin(3x + yz)$$

$$f_{xxy} = -9z\cos(3x + yz)$$

$$f_{xxyz} = -9\cos(3x + yz) + 9yz\sin(3x + yz)$$

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Total Derivatives

Total Derivatives

$$Jika\ z = f(x, y),\ x = g(t),\ y = h(t)$$
$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t}$$